Game of Gorgons

Kyle Burke
joint work with Craig Tennenhouse

Florida Southern College

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Talk Plan

- Introduce Gorgons
- Play Gorgons
- Values and Properties
- Sprouts 2024's Game
- Conclusions and Open Problems

Gorgons

Gorgons is a Combinatorial Game

- Two players alternate turns
- No randomness
- No hidden information
- Normal Play: If you can't make a move, you lose. I.e. Last player to move wins.
- Examples: Chess, Checkers

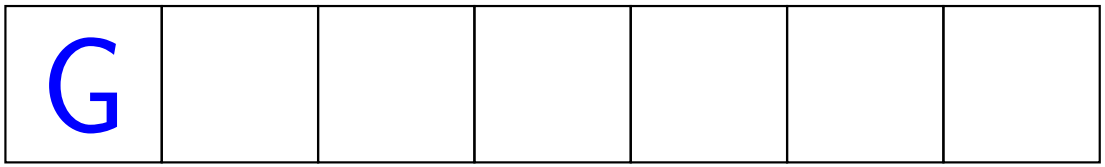
Gorgons

Gorgons:

- Position: 2-D grid. Each space:
  - Empty, or
  - Stone block, or
  - Gorgon (Blue or Red, the two players), or
  - Gorgon trapped in stone.
- Each turn:
  - Pick (non-stone) Gorgon, then
  - Pick a direction to face
    - Cardinal 8 directions
    - Direction can't be blocked by stone or edge of board
  - Turn furthest space in that direction to stone, then
  - Move in that direction (optional)
- If you can't take a turn with any Gorgon, you lose. (I.e., none of your non-stone Gorgons can take a turn.)

Gorgons

Move Examples:



Gorgons

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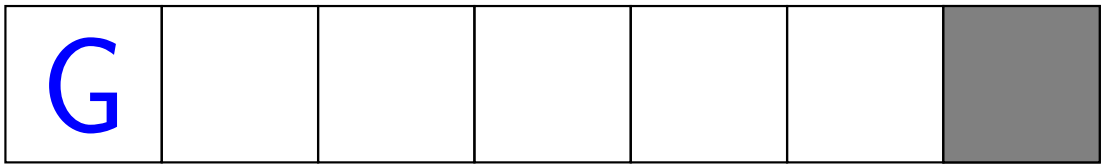

Gorgons

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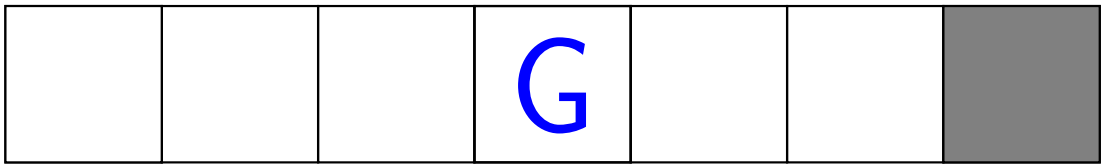

Gorgons

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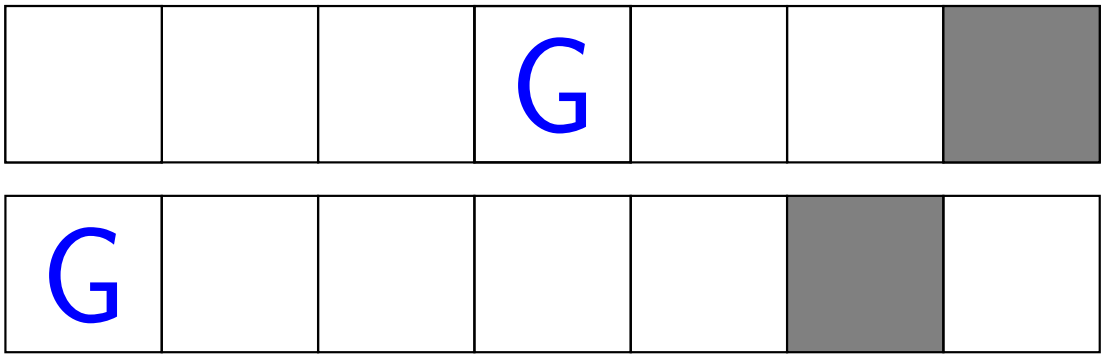

Gorgons

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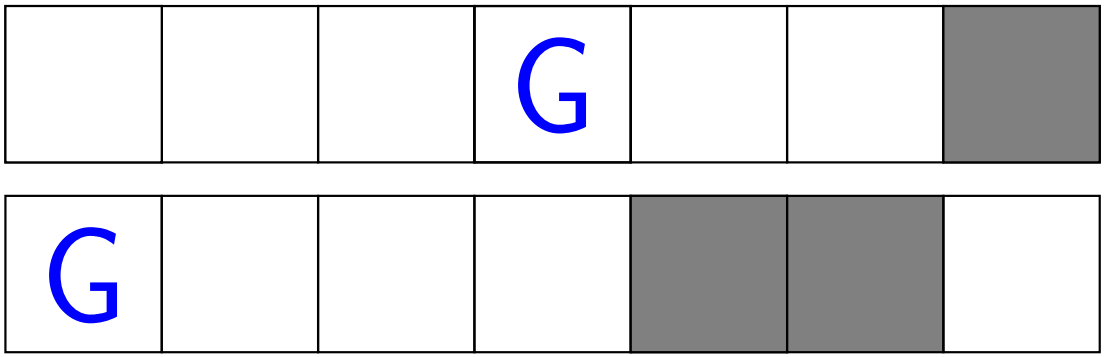

Gorgons

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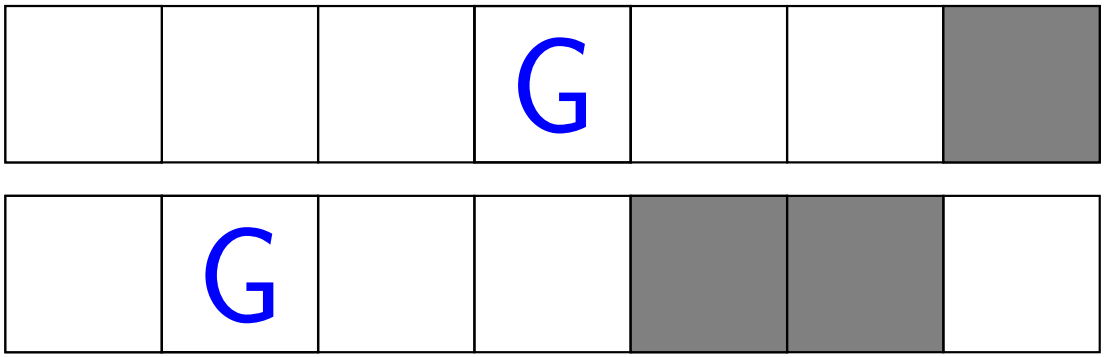

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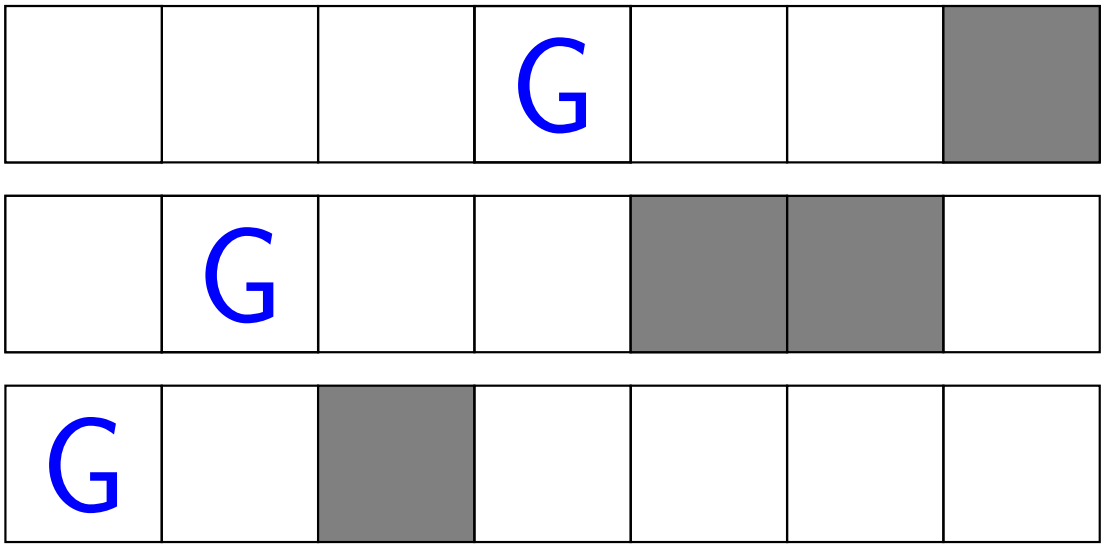

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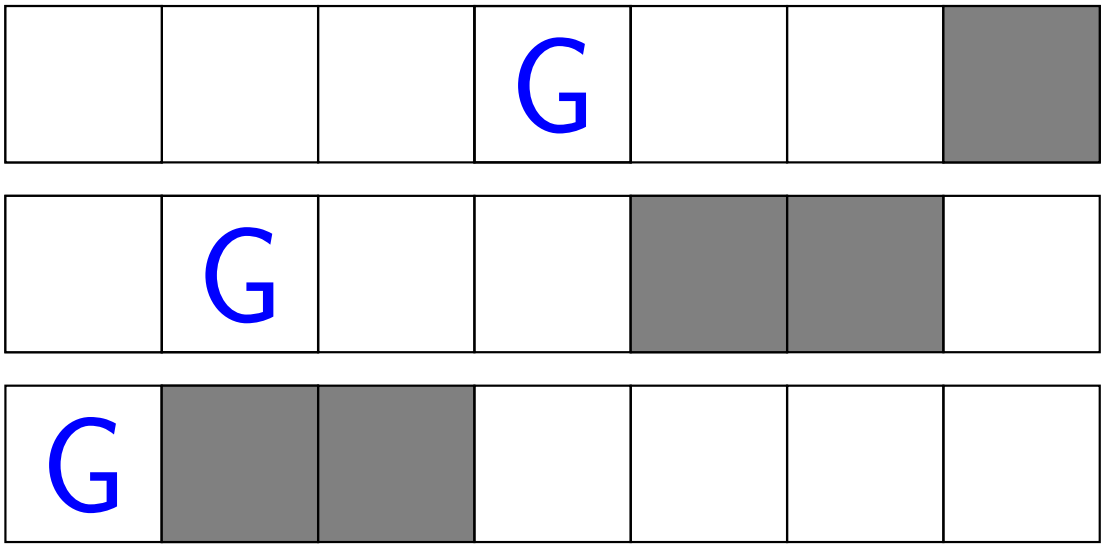

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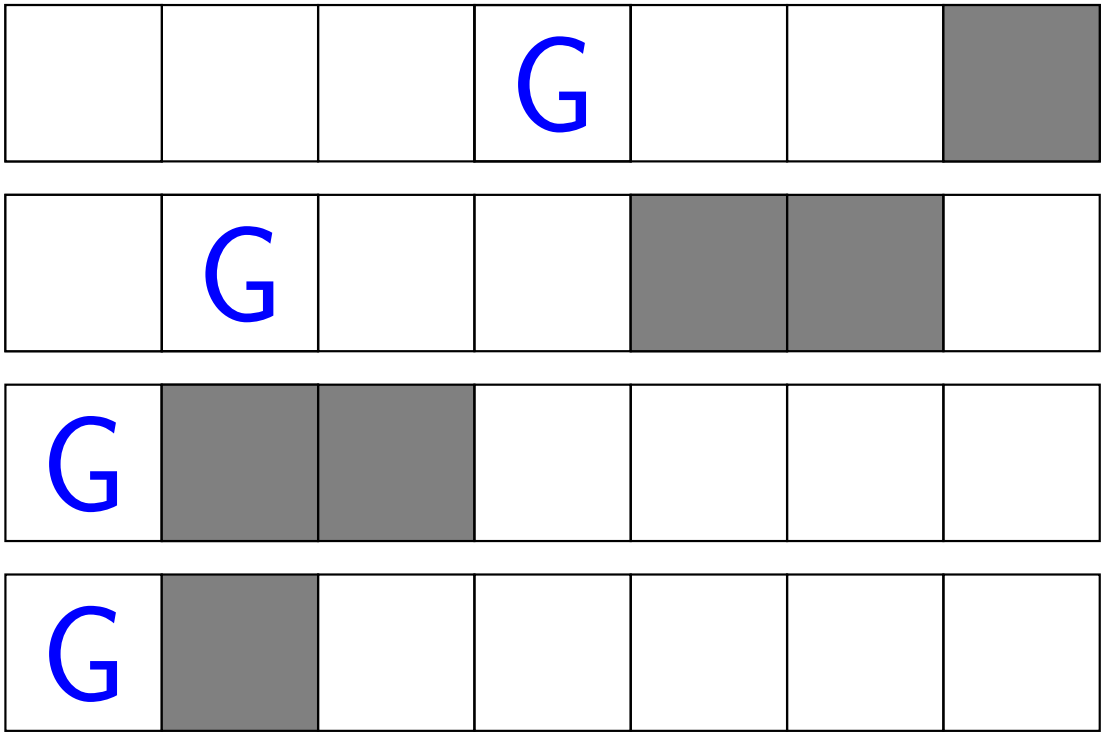

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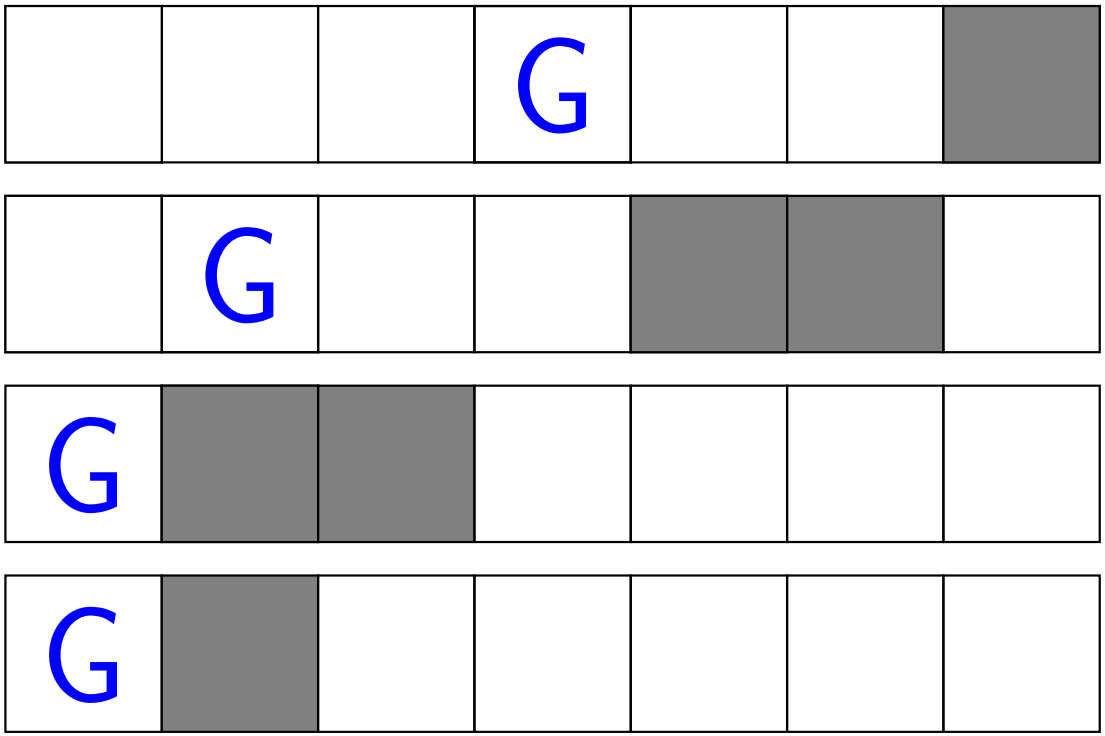

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Gorgons

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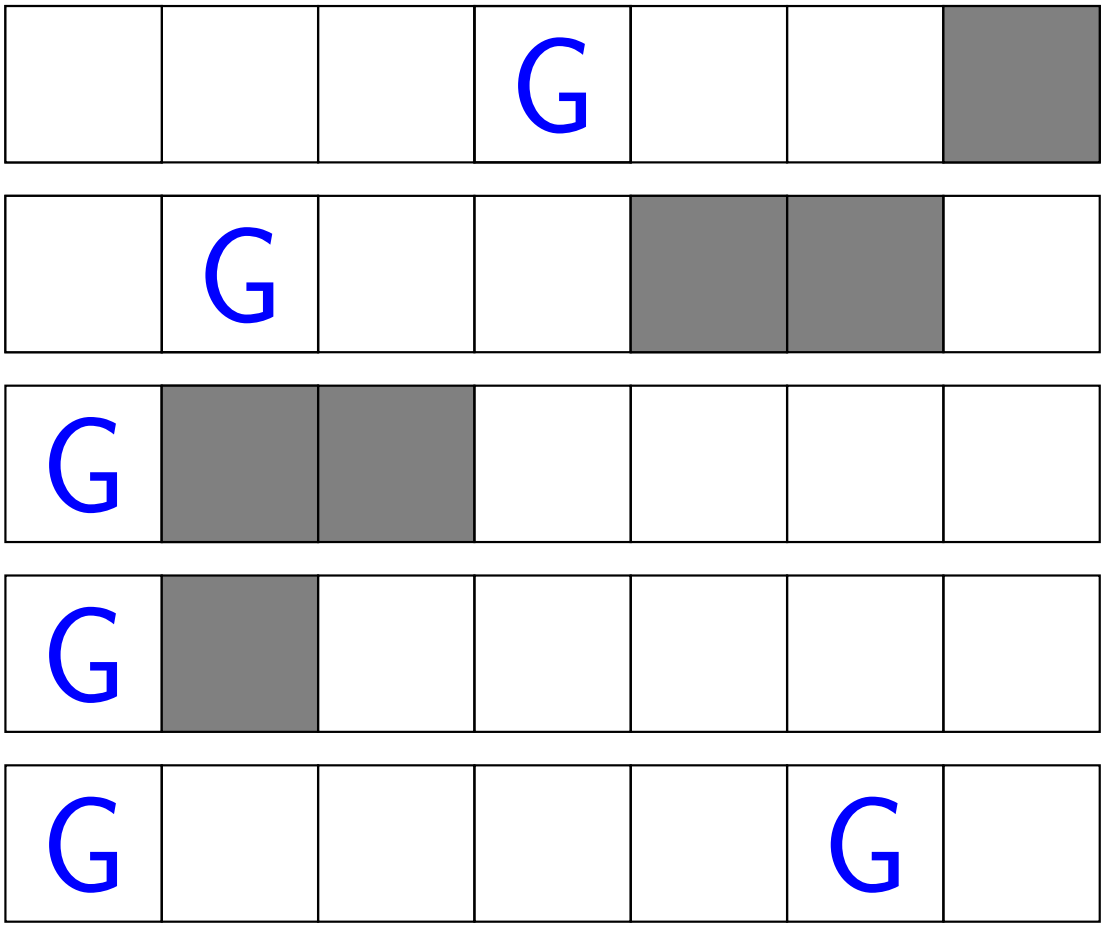

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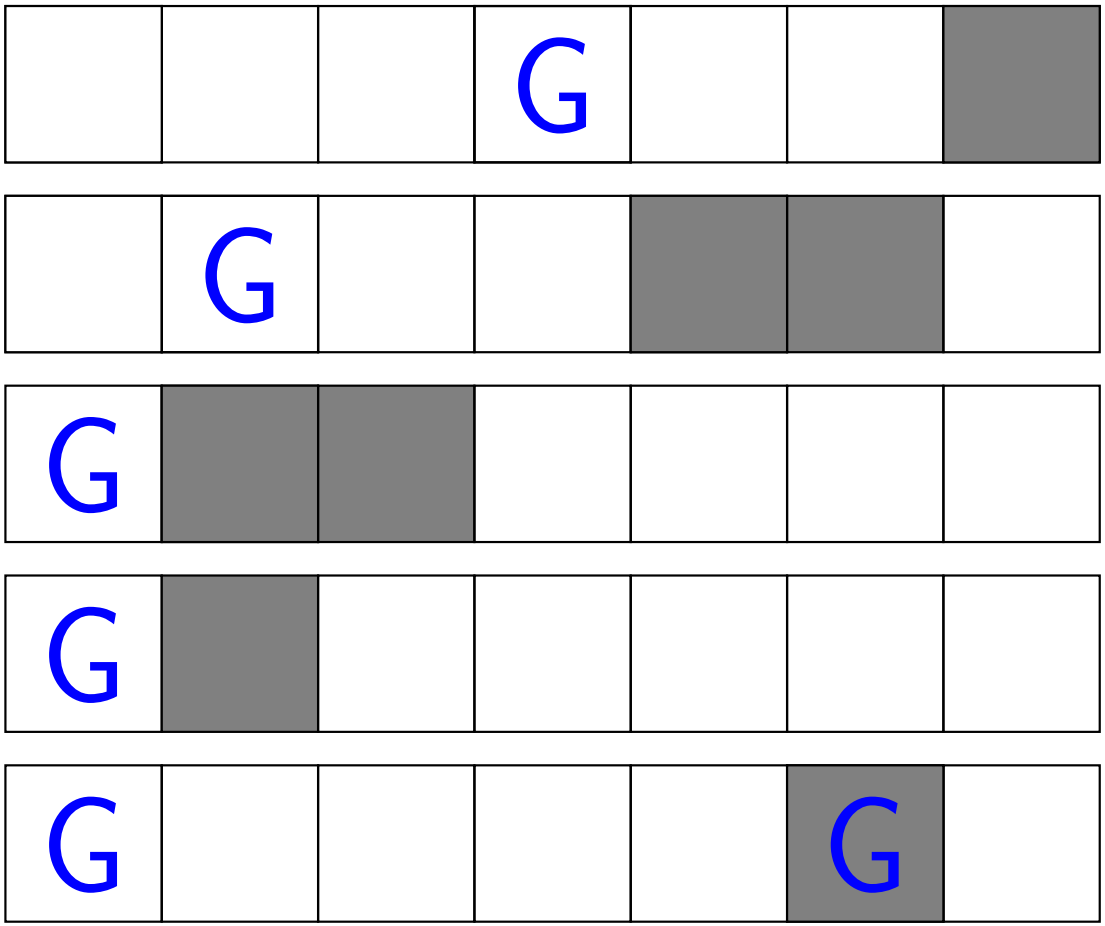

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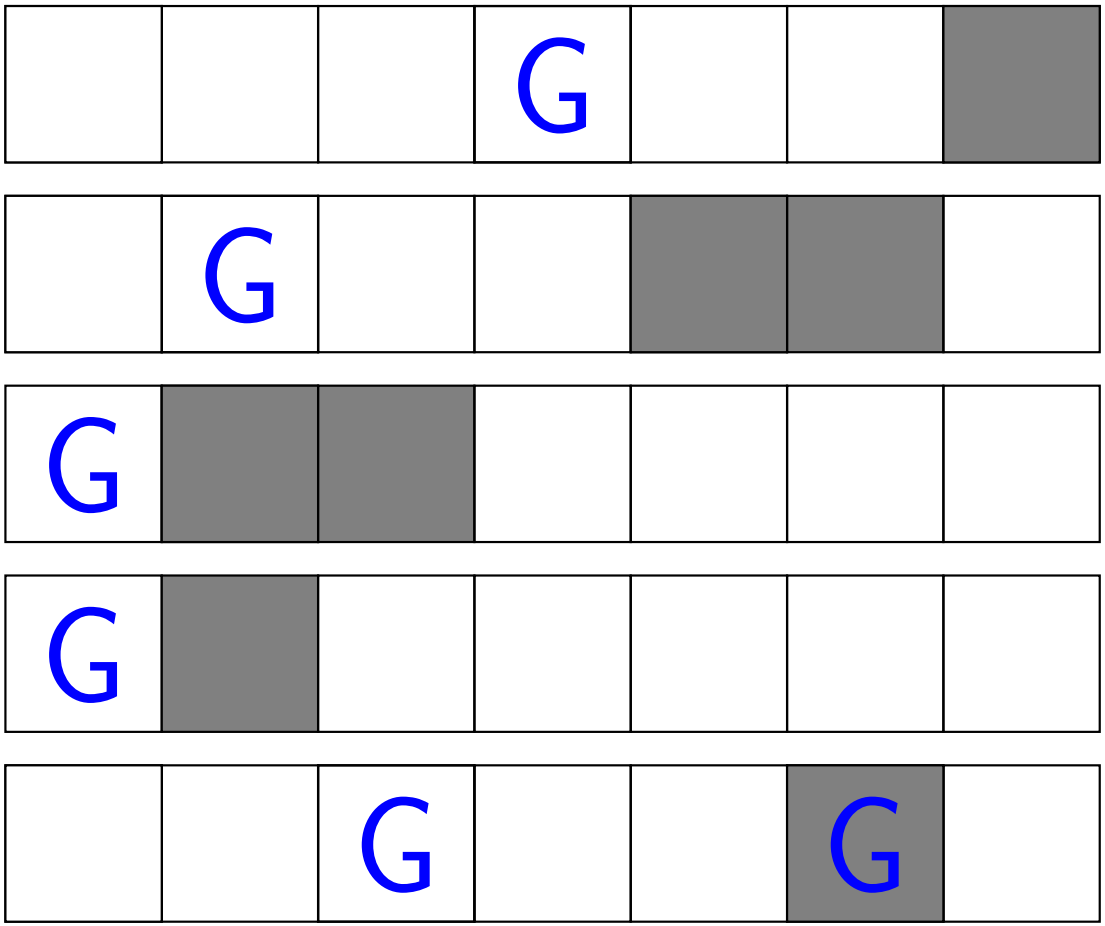

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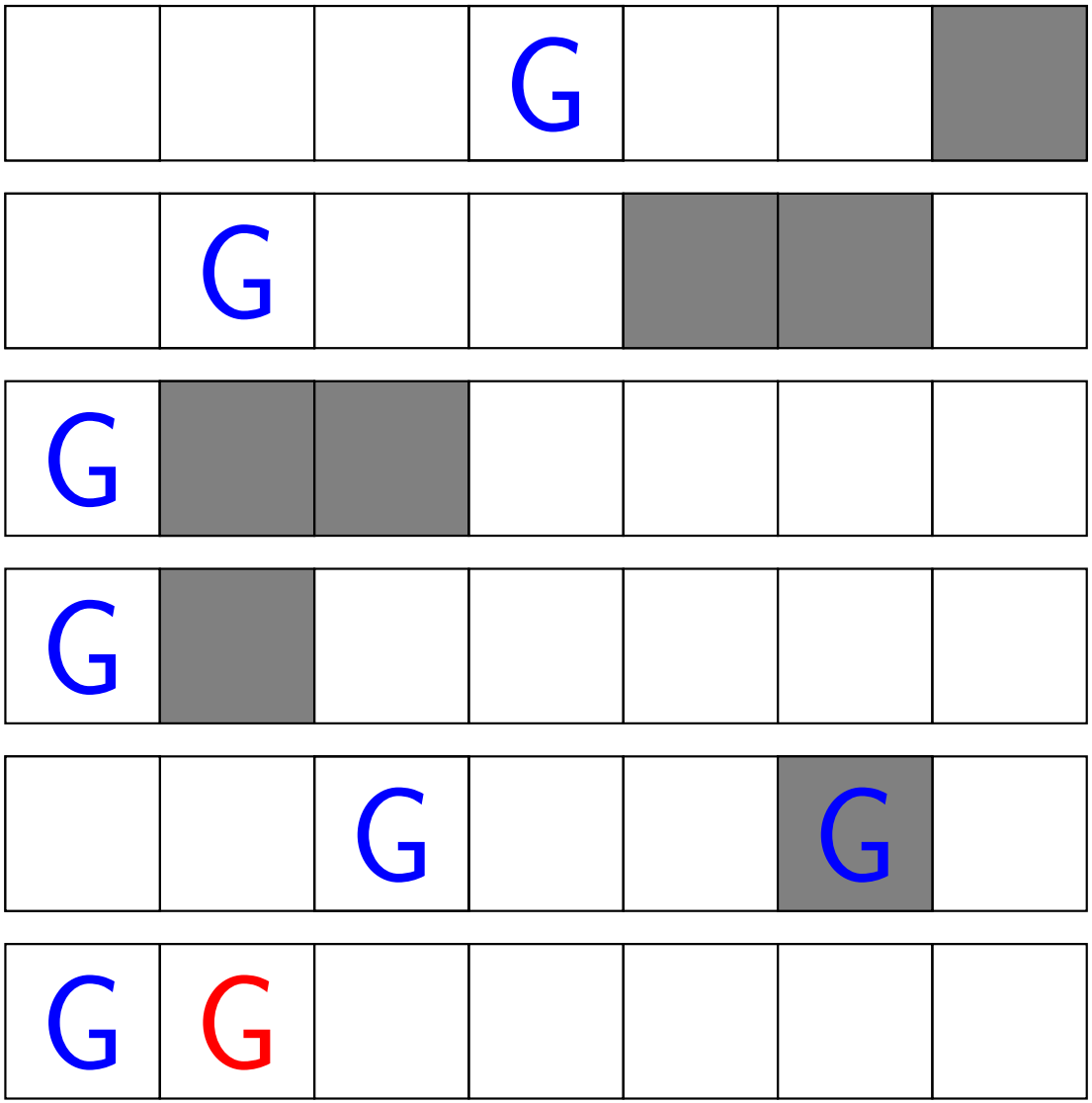

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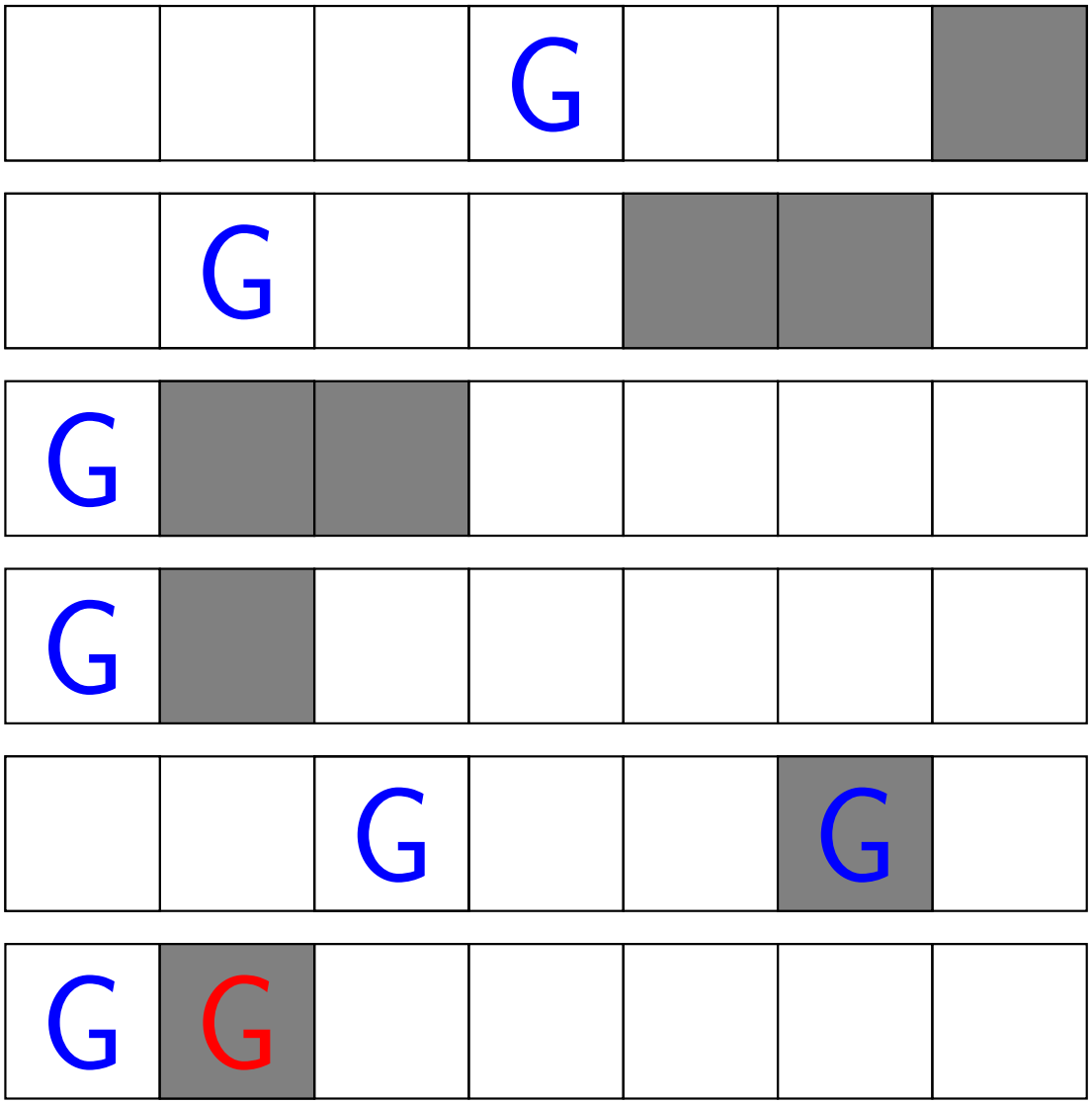

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Gorgons

Let's play:
`http://kyleburke.info/DB/combGames/gorgons.html`

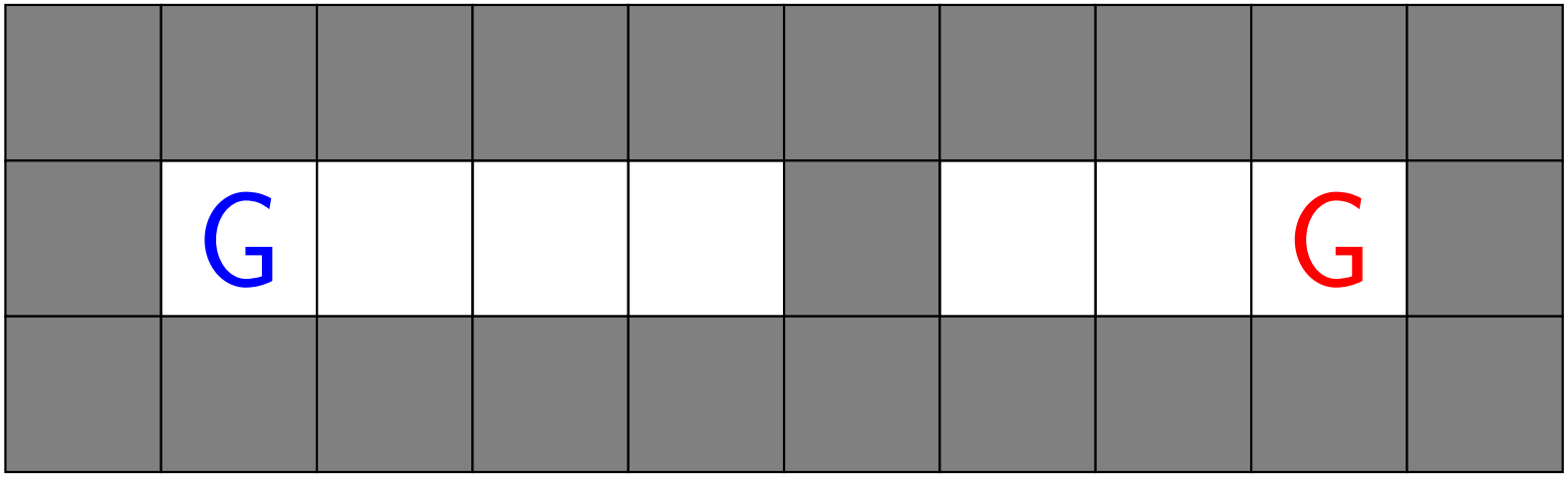
Gorgons

Gorgons Implementation:

- Language: JavaScript/CSS (All graphics are in SVG.)
- Uses `combinatorialGames.js` (`http://kyleburke.info/DB/combGames/combinatorialGames.js`)
- I have some pre-programmed "AI" players that just traverse the game tree to a given depth. (These are bad! I need to implement MCTS.)
- Many other games use the package: (`http://kyleburke.info/DB/combGames/`). (Senior Projects!)
- Easy to add more games!

Gorgons Values and Properties

Who wins? (means: Who has winning strategy, Blue or Red?)

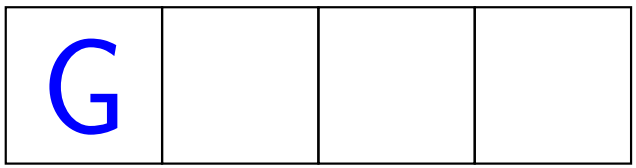


Blue has three moves, Red has two moves.
What if Red goes first? Blue still wins!
Left part: 3 moves; Right part: 2 moves.
Values:

- 3 moves for Blue: 3
- 2 moves for Red: -2
- $3 - 2 = 1$ Positive, so Blue can always win.

Gorgons Values and Properties

More formally, recursively define position
$G = \{$ Left options $|$ Right options $\}$

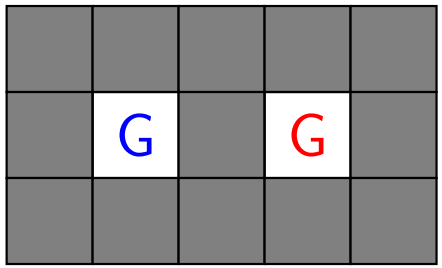
- 

$= 3 = \{\ 2 \mid\ \}$
- Options: 

- $\{\ 2, 2, 2 \mid\ \} = \{\ 2 \mid\ \} = 3$
- In general, $\{\ n \mid\ \} = n + 1$ (For natural numbers n.)
- Also, $\{\ \mid -n\ \} = -n - 1$.
- Goal: at end of game, have more free space than your opponent!
- What is zero?

Gorgons Values and Properties

Zero

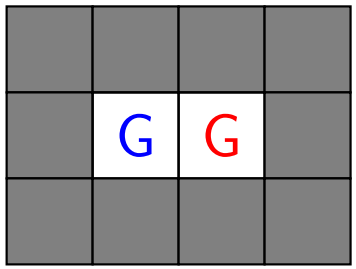
- The basic form of zero is $\{ \ | \ \}$.
- E.g.: 

$= \{ \ | \ \} = 0$
- Who wins on $\{ \ | \ \}$? A: Whoever goes *second*.
- What about $2 + -2$? Who wins on that? A: Same! Whoever goes second.
- Thankfully, $2 - 2 = 0$.
- (All second-player-winnable games are equal to 0.)

GORGONS Values and Properties

What about $\{ 0 \mid 0 \}$?

- Is there a GORGONS position equal to $\{ 0 \mid 0 \}$?
- E.g.:

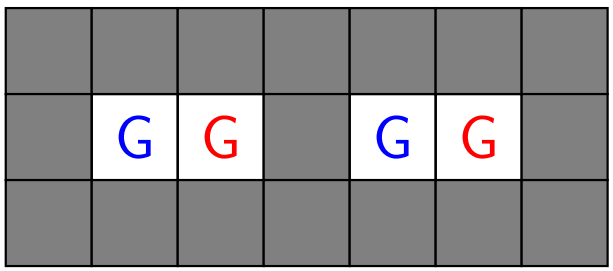


- Who wins on $\{ 0 \mid 0 \}$? A: Whoever goes *first*.
- Neither positive nor negative (nor zero).
- We use another symbol: $\{ 0 \mid 0 \} = *$.

Gorgons Values and Properties

What about $* + *$?

- Who wins $* + *$?

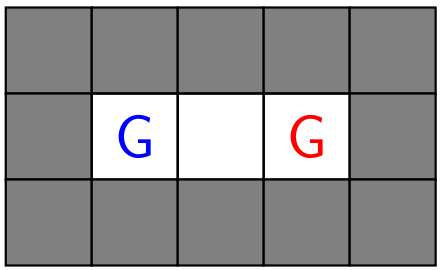


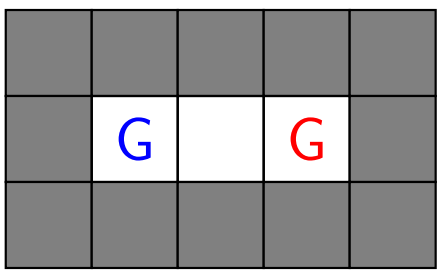
- Winner: whoever goes second.
- $* + * = 0$ (Stars cancel each other out.)

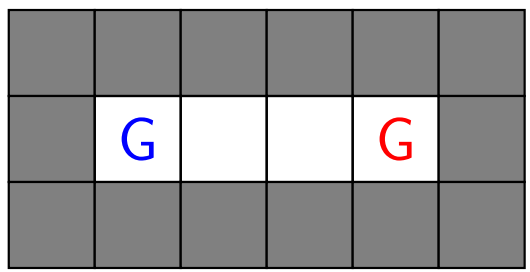
Gorgons Values and Properties

Switches

- What is the value of this position?



- $\{ 1 \mid -1 \} = \pm 1$, known as a *switch*.
- This is a hot game: players *want* to play here because it earns them "points".
- $\{ k \mid -k \} = \pm k$ (when $k \geq 0$)
- 

$+$ 

$= \pm 1 \pm 2$

Gorgons Values and Properties

What is the habitat of Gorgons? What other values are in Gorgons?

- Integers ✓
- $*$ ✓
- Switches ✓
- $*2 = \{ 0, * \mid 0, * \}$?
- Other nimbers? $(*3, *4, *5, \ldots)$
- Fractions? $\frac{1}{2} = \{ 0 \mid 1 \}$ $\frac{1}{4} = \{ 0 \mid 1/2 \}$
- More combinatorial game values? (Check out our book: Playing with Discrete Math)[1]

[1] http://kyleburke.info/CGTBook.php

Gorgons at Sprouts 2024

Sprouts 2024:

- Sprouts: Combinatorial Games conference focused on undergraduates. (`http://kyleburke.info/sprouts/`)
- 2017, 2018, 2019, ~~2020~~ virtual: 2022, 2023
- Sprouts 2024: will be online
  - Saturday, April ?
  - Zoom talks. (Many 15-minute slots.)
  - Conference Human Tournament
  - Conference AI Tournament (`http://kyleburke.info/sprouts/sprouts2024/sprouts2024ComputerTournament.php`)
  - Last year: two AI players.
- I'm hoping to have an in-person tournament the (Friday) afternoon before.

Conclusions

GORGONS:

- Is a moderately fun game. (Probably not as fun as AMAZONS.)
  - Might add some initial blocks to the starting board.
- Has lots of game values.
- The tournament system is already ready to go!
- There is still more to be done, though...

Future Work

GORGONS:

- There are likely lots of other values attainable in GORGONS.
- We don't yet know how difficult it is to computationally determine the winner.
  - Sometimes games are very easy (NIM, BRUSSELS SPROUTS): we can determine the winner in polynomial time, or
  - Often, games are hard (FLAG COLORING, AMAZONS): no known polynomial time algorithm exists.
- My guess: GORGONS is PSPACE-hard.
- To show that, we would need a reduction, from another PSPACE-hard game R to GORGONS.
- That is a function, f: $R \to$ GORGONS, where
  Left wins x going first $\Leftrightarrow$ Left wins $f(x)$ going first.

Thank you!

Thank you!

Please come to Sprouts in April!

Please code up a player for GORGONS!