

# Towards an Impartial Short Tafl Variant

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- Motivation

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- Background on Tafl

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- Some possible rulesets

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- What's next?

# Motivation

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It starts with a trip to the National Museum in Edinburgh

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It starts with a trip to the National Museum in Edinburgh



What game is this?

It starts with a trip to the National Museum in Edinburgh



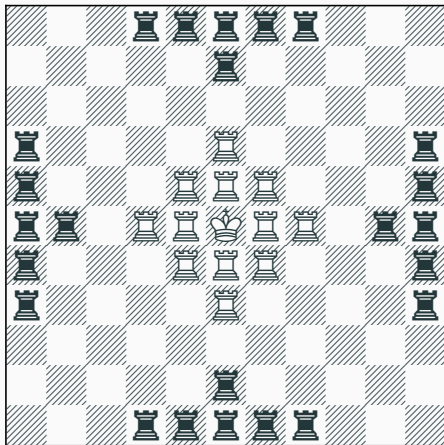
What game is this?  
Probably Hnefatafl

# Tafl games

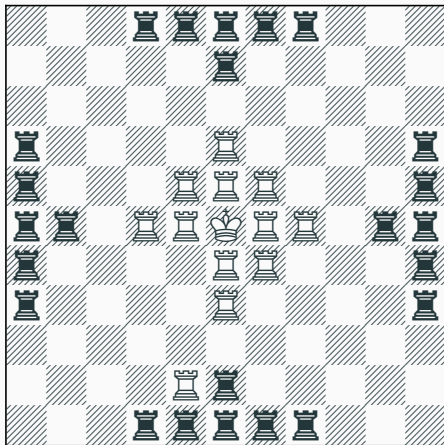
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What are Tafl games?

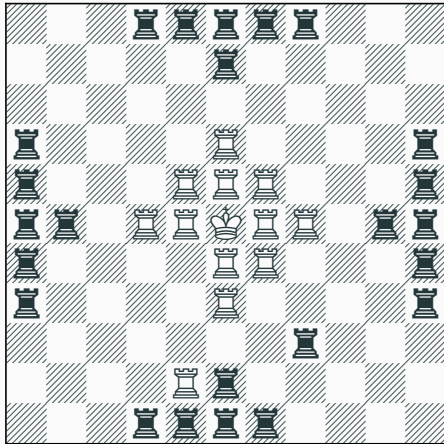
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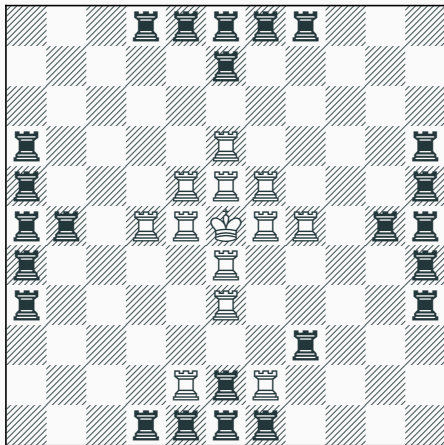
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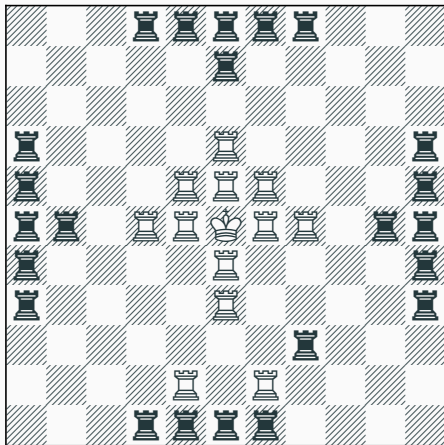


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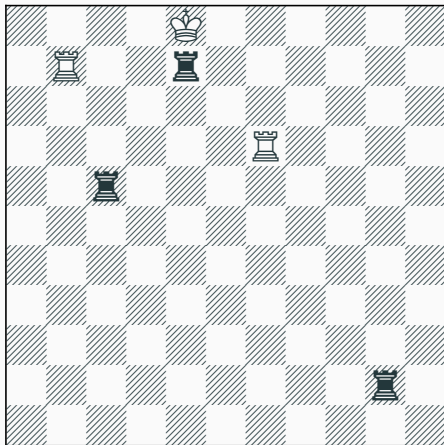




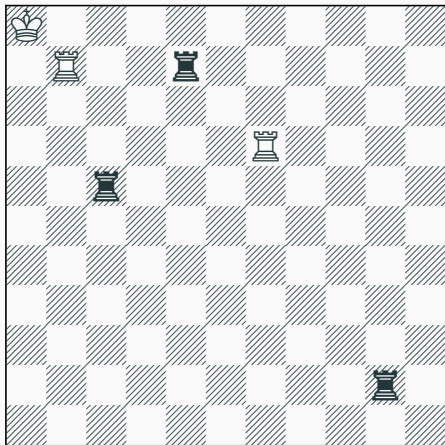
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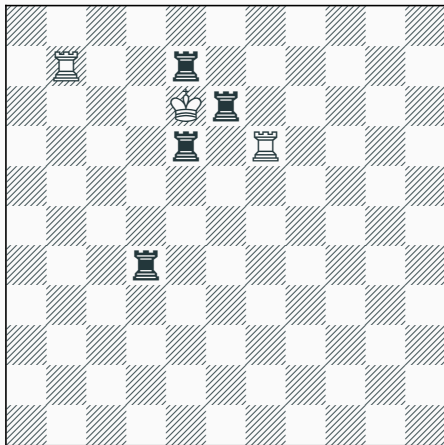
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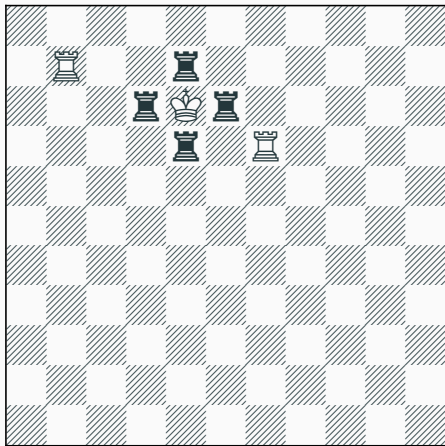
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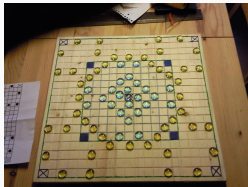


Many variants

Many variants



Many variants



Always loopy



Many variants



Always loopy

Always partisan





This looks impartial!



This looks impartial!  
Can we analyze it?

# Rulesets

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Game ends when king can't move (exit or captured)

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Only analyzed with king and one soldier

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King moves N/W



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Possible soldier movement mechanics

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- Any direction, ending closer to K
- Only N/W

Game ends when king can't move (exit or captured)

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King moves N/W

Possible soldier movement mechanics

- Any direction, ending closer to K
- Only N/W
- Only S/E

Game ends when king can't move (exit or captured)

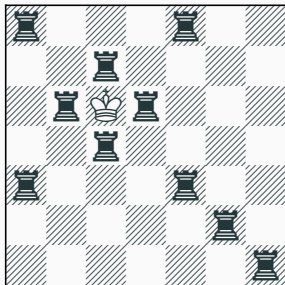
Only analyzed with king and one soldier

King moves N/W

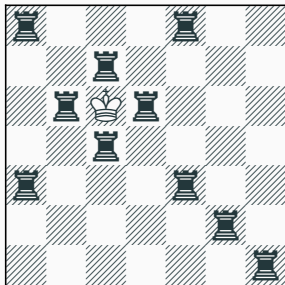
Possible soldier movement mechanics

- Any direction, ending closer to K
- Only N/W
- Only S/E
- Stay stationary

## End closer to K

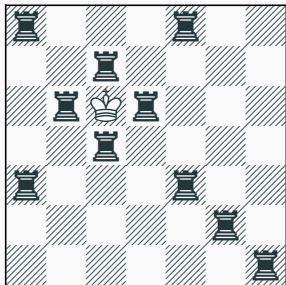


## End closer to K

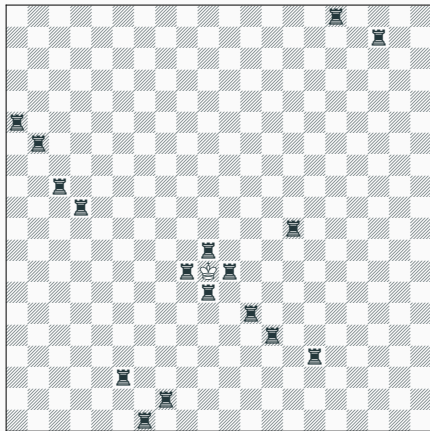


Looks simple, but

# End closer to K

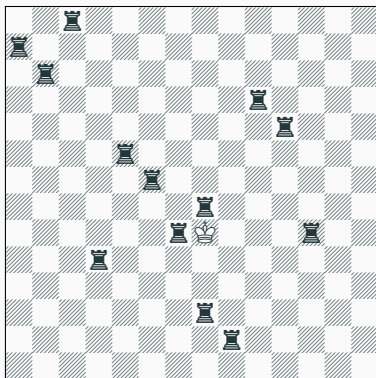


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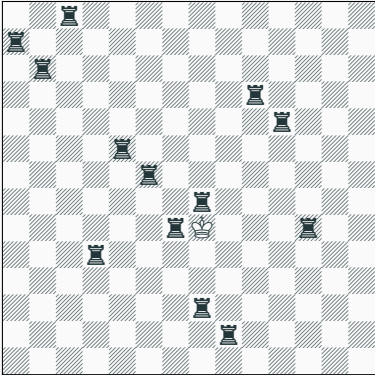




# Soldier moves N/W

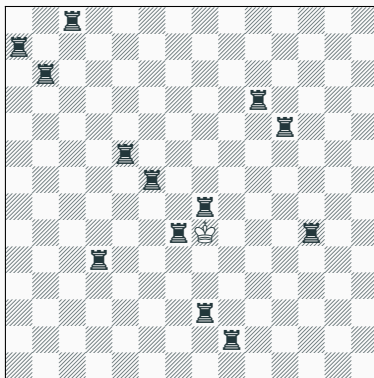


# Soldier moves N/W



This is really just blocking nim with a strange blocking mechanic

## Soldier moves N/W



This is really just blocking nim with a strange blocking mechanic  
Not analyzed

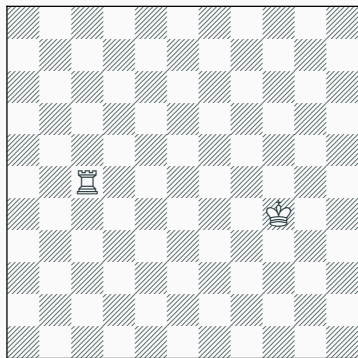
Let S move S/E, but don't pass K

Let S move S/E, but don't pass K

Why would a soldier remove themselves from the coup?

Let S move S/E, but don't pass K

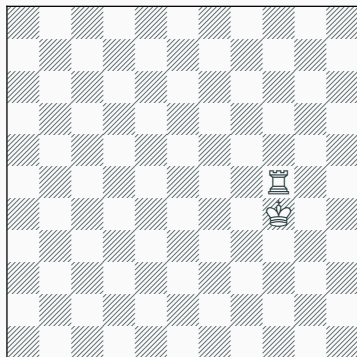
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K can pass soldier

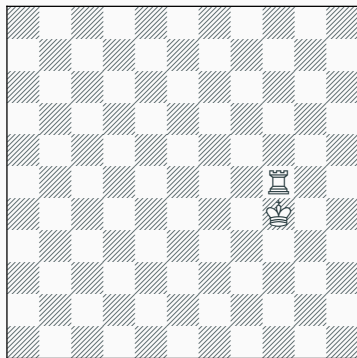
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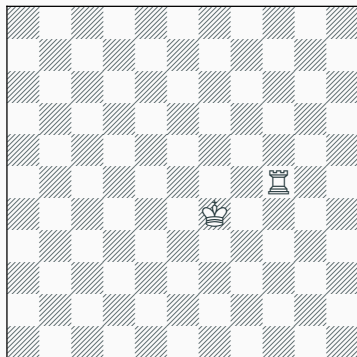


K can pass soldier



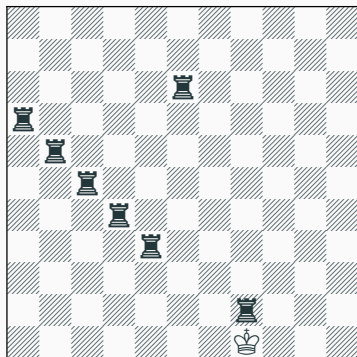
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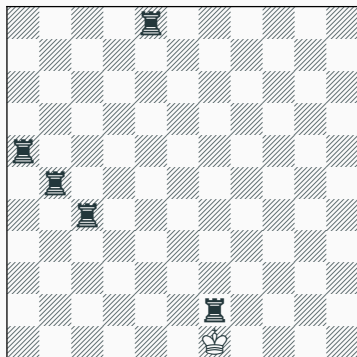


K can pass soldier

## $\mathcal{P}$ -positions



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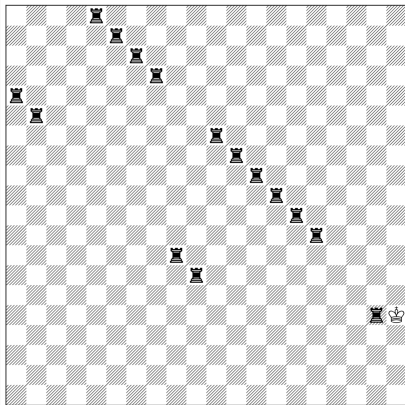


$\mathcal{P}$ -positions

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Looks modular relative to distance from main diagonal

## $\mathcal{P}$ -positions



Assume  $a \leq b$  and  $|a - b| = r$

**Theorem**

$(K = (a, b), S = (c, d)) \in \mathcal{P}$  iff

Assume  $a \leq b$  and  $|a - b| = r$

### Theorem

$(K = (a, b), S = (c, d)) \in \mathcal{P}$  iff

$$K = (0, b) \Rightarrow S = (0, b - 1)$$

$$K = (a, a) \Rightarrow S \in \{(a - 1, a), (a, a - 1), (k, k) : k < (a - 1)\}$$

$$K = (a - 1, a) \Rightarrow S \in \{(a - 1, a - 1), (k - 1, k) : 1 \leq k \leq (a - 2)\}$$

$$K = (a - 2, a) \Rightarrow S \in \{(a - 2, a - 1), (k - 2, k) : 2 \leq k \leq (a - 2)\}$$

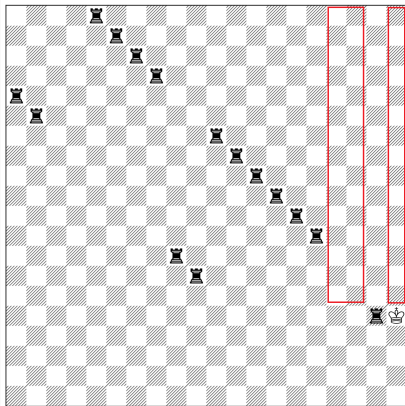
Otherwise  $S \in \{(a, b - 1)\} \cup$

$$\{(a - 1 - 2kr - i, b - 1, 2kr - i) : (r - 1) \leq i \leq 2r, k \geq 0\} \cup$$

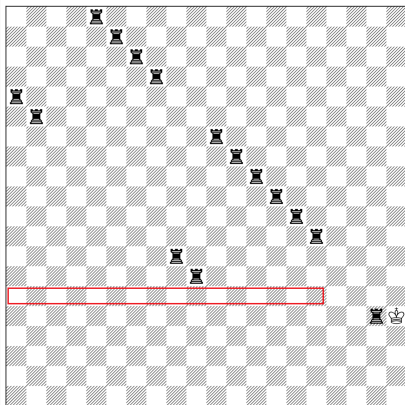
$$\{(a - 1 - 2kr - i, b - 1 - 2(k + 1)r - i) : 1 \leq i \leq (r - 2), k \geq 0\}$$



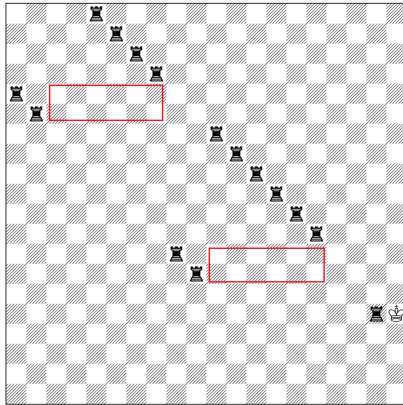




*K* moves to  $(a, a)$ , removing soldier from play



*K* moves West next to *S*



$K$  moves West to diagonal from  $S$ , or off-diagonal as needed.

If King moves all responses to  $\mathcal{P}$  are analogous  $\square$

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What if the soldiers are stationary?

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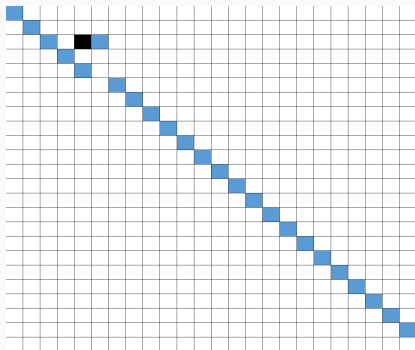
What if the soldiers are stationary?

Variant of Blocking Nim

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Variant of Blocking Nim





If King moves all responses to  $\mathcal{P}$  are analogous  $\square$

What if the soldiers are stationary?

Variant of Blocking Nim

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	0	3	2	5	4	7	6	9	8	11	10	13	12	15	14	17	16	19	18	21	20	23	22
2	3	0	1	x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
3	2	1	0	4	6	5	8	7	10	9	12	11	14	13	16	15	18	17	20	19	22	21	24
4	5	6	7	0	1	2	3	10	11	8	9	14	15	12	13	18	19	16	17	22	23	20	21
5	4	7	6	1	2	0	9	11	3	12	8	10	16	17	18	13	14	15	21	23	19	24	20
6	7	4	5	2	3	8	0	1	12	13	14	9	10	11	17	19	15	20	16	18	24	25	26
7	6	5	4	3	8	9	1	0	2	14	13	15	11	10	12	20	21	22	23	16	17	18	19
8	9	10	11	5	7	3	4	2	0	1	15	6	17	16	19	12	13	14	22	24	18	26	25
9	8	11	10	6	12	4	5	13	1	0	2	3	7	18	20	14	22	21	15	17	25	16	27
10	11	8	9	7	13	12	14	4	5	2	0	1	3	6	21	22	20	23	24	25	19	16	17
11	10	9	8	12	14	13	15	5	6	3	1	0	2	4	7	21	23	24	25	26	27	28	17
12	13	14	15	8	9	10	11	6	7	4	3	2	0	1	5	23	24	25	26	27	28	29	30
13	12	15	14	9	10	11	16	17	18	6	4	5	1	0	2	3	7	8	27	28	26	30	29
14	15	12	13	10	11	16	17	18	19	7	5	4	6	2	0	1	3	9	8	29	30	27	28
15	14	13	12	11	16	17	10	19	20	18	7	8	4	3	1	0	2	5	6	9	29	31	32
16	17	18	19	13	15	14	12	20	21	22	23	24	5	7	3	2	0	1	4	6	8	9	10
17	16	19	18	14	20	15	13	12	22	21	24	23	9	5	4	6	1	0	2	3	7	8	11
18	19	16	17	15	21	20	22	14	13	23	25	26	24	8	6	4	5	2	0	1	3	7	9
19	18	17	16	20	22	21	23	15	14	24	26	25	27	28	8	5	4	3	1	0	2	6	7
20	21	22	23	16	17	18	19	24	15	25	27	28	26	29	9	7	6	4	3	2	0	1	5
21	20	23	22	17	18	19	24	16	25	15	28	27	29	26	11	8	9	6	5	4	1	0	2
22	23	20	21	18	19	24	25	26	16	17	29	30	28	27	31	9	8	7	10	5	4	2	0
23	22	21	20	19	24	25	18	27	17	16	30	29	31	32	26	10	11	12	7	8	5	3	1

## Next steps

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Fun game!

Fun game! What's next?

Fun game! What's next?

Multiple soldiers?

Fun game! What's next?

Multiple soldiers?

Values for main ruleset?

Fun game! What's next?

Multiple soldiers?

Values for main ruleset?

How to generalize the board to a graph?

Fun game! What's next?

Multiple soldiers?

Values for main ruleset?

How to generalize the board to a graph?

Who cares?



Fun game! What's next?

Multiple soldiers?

Values for main ruleset?

How to generalize the board to a graph?

Who cares? Seriously, tell me who cares.